Calculus Warm Up #4-3

Same problem...
The radius of a sphere is increasing at a constant rate of 0.04 cm/sec.

At the time when the volume and the radius are increasing at the same rate, what is the radius?

HW questions: p. 598, # 23 - 37 odd

27) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ Alternating Series Test

$\lim_{n\to\infty} \frac{1}{\ln n} = 0$ ✓

$\frac{1}{\ln(n+1)} \le \frac{1}{\ln n}$ for $n \ge 3$ ✓

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ converges

Now check:

$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{\ln n} \right| = \sum_{n=1}^{\infty} \frac{1}{\ln n}$ compare to $\sum \frac{1}{n}$

(If you use Limit Comparison Test you get $\lim_{n\to\infty} = 0$, so test is inconclusive)

Direct Comparison Test

$\frac{1}{n}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
$\frac{1}{\ln n}$	$\cancel{0}$	1.44	0.91	0.72	0.62

$0 \le \frac{1}{n} \le \frac{1}{\ln n}$

Since the smaller diverges, $\sum \frac{1}{n}$ (divergent p-series $p=1$), $\sum \left(\frac{1}{\ln n}\right)$ also diverges.

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$ converges conditionally since

$\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{\ln n} \right|$ diverges

37) $$\sum_{n=1}^{\infty} \frac{\sin \frac{\pi}{2}(2n-1)}{\sqrt{n}} = \frac{\sin\frac{\pi}{2}}{1} + \frac{\sin\frac{3\pi}{2}}{\sqrt{2}} + \frac{\sin\frac{5\pi}{2}}{\sqrt{3}} + \frac{\sin\frac{7\pi}{2}}{\sqrt{4}} + \cdots$$

$$= \frac{1}{1} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

Alternating Series Test

$$\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}} \checkmark$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0 \checkmark$$

converges

Checking $\displaystyle\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

divergent
p-series
$p = \frac{1}{2} < 1$

$\therefore \displaystyle\sum_{n=1}^{\infty} \frac{\sin\frac{\pi}{2}(2n-1)}{\sqrt{n}}$ converges conditionally

since $\displaystyle\sum_{n=1}^{\infty} \left| \frac{\sin\frac{\pi}{2}(2n-1)}{\sqrt{n}} \right|$ diverges

43) $$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} = \overset{n=0}{\frac{1}{1}} - \frac{1}{6} + \frac{1}{120} - \frac{1}{5040}$$

$|R_2| \leq a_3$

$a_3 \approx 0.000198$

$$S \approx S_2 = \sum_{n=0}^{2} \frac{(-1)^n}{(2n+1)!}$$

$$\approx 0.842$$

$S = \sin 1$
≈ 0.84147

C: ,

Estimating the sum of a Converging Alternating Series:

When $a_{n+1} \leq a_n$, using the partial sum, S_N to estimate $S = \sum a_n$

The accuracy of the estimate:

$$|S - S_N| = |R_N| \leq a_{N+1}$$

N = Term # of the last term in the series included in the partial sum

R_N = Remainder of the Series (sum of the remaining terms)

Does the series converge? If so, approximate the sum using the first 6 terms. How accurate is your result?

$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\frac{1}{n!}\right)$$

$$\lim_{n\to\infty} \frac{1}{n!} = 0 \checkmark$$

$$= \frac{1}{1} - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720} + \cdots$$

$$\boxed{\frac{1}{(n+1)!} \leq \frac{1}{n!} \checkmark}$$

Series converges by the Alternating Series Test.

Approximate the sum using the first 6 terms.
How accurate is your result?

$$\sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{1}{n!}\right) \qquad \text{Since } \frac{1}{(n+1)!} \leq \frac{1}{n!}$$

$$S \approx S_6 = \frac{1}{1} - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \frac{1}{720}$$

$$S_6 = \frac{91}{144} \approx 0.6319$$

How accurate is your result?

The accuracy of the estimate:

$$|S - S_N| = |R_N| \leq a_{N+1}$$

$$\sum_{n=1}^{\infty} (-1)^{n-1}\left(\frac{1}{n!}\right)$$

$$S \approx S_6 = \frac{91}{144} \approx 0.6319$$

Alternating Series Remainder:

$$|R_6| \leq a_{6+1} \qquad a_7 = \frac{1}{7!} = \frac{1}{5040}$$

$$|R_6| \leq \approx 0.0002 \qquad \approx 0.0002$$

$\therefore$ The actual Sum, S, is $\approx$

$$0.6319 - |R_6| \leq S \leq 0.6319 + |R_6|$$

$$0.6317 \leq S \leq 0.6321$$

Classwork:

2002 FR - B, # 6

(Blue)

6a) 5 km

b) -6 km/hr.

c) $\frac{17}{5}$ radians/hr.

HW: p. 598, # 23 - 45 odd

(day 2)

Group quiz Friday over 10.1 - 10.4